Question #1

By installing a $120 thermostat that reduces the temperature setting at night, a family hopes to cut its annual bill for heating oil by 10 percent, and thereby recover the cost of the thermostat in fuel savings after 2 years. What was the family's annual fuel bill before installing the thermostat?

Answer #1

The family wants to save $120 over a period of two years. That means they want to save $60 a year, right? The problem also states they want to cut their yearly bill by 10 percent. Well, the two paragraphs above tell you what you need to know…that $60 is 10 percent of the total bill. Let's set up a proportion and solve for the total bill:

- $\frac{60}{x} = \frac{10}{100}$
- Cross multiply: $60 = 0.10x$
- Divide both sides by 0.10
- $600 = x$
- The total yearly bill is $600.

Let's check the answer. The family wants to save $120 over two years by saving 10 percent each year. Well, 10 percent of $600 is $60 so the new bill will be $540 per year. Over two years, that adds up to $120 in savings.

Question #2

Gary and Andrea have both agreed to mow half of a lawn. This lawn is a rectangle, 25 feet by 45 feet. Their lawn mower cuts a 2 feet wide path. If Andrea starts at a corner and makes a path around the outside, approximately how many times should she go around to cut one half of the lawn?

Answer #2

The total area is $25 \times 45 = 1125$. Half the area is 562.5. The area cut during the first circuit will be the total area minus area left after first cut: $(25 \times 45 - 21 \times 41) = 264$ square feet. Likewise, during the second circuit, the area cut will be: $(21 \times 41 - 17 \times 37) = 232$. The total area cut so far is $264 + 232 = 496$. The area that Andrea still has to cut is $562.5 - 496 = 66.5$ sq. ft. If we do a complete third circuit, the area cut would be $(17 \times 37 - 13 \times 33) = 200$. So, Andrea has $66.5/200 = .3325$ (approx. 1/3) of the third circuit to make. Andrea does approximately $2 + (1/3)$ circuits to cut half the lawn.

Question #3

A factory owner knows that some of her expenses are going to occur regardless of how many items she makes. These are fixed costs and total $3000 per week. This means that if she makes no sales she still has to pay the $3000 per week. The remaining expenses are variable costs. The most obvious variable cost is the stock she buys, but there are others such as labor. She has found the variable costs are almost directly related to sales.

In a week in which she made 500 items, variable costs were $7500. The items are all made to order and so are all sold. They sell at $20 each.

Question #3 continued

1) Draw up a table to show some values for income to the shop, ***i***, for different numbers of items sold, ***n***.

2) Draw up a table for the total cost for different numbers of items sold. These are the fixed costs added to the variable costs. Remember that the variable costs are directly related to the number sold.

3) The difference between expenses and income is the factory owner's profit.

If she made no sales she would still have to pay the fixed costs, so she would have a very large loss. If she made many sales, she would have easily covered her fixed costs and made a good profit. In a week in which she sold 500 items did she make a profit or a loss? In a week in which she sold 1000 items did she make a profit or a loss?

On the same pair of axis draw the two graphs to show income related to the number of items sold and expenses related to the number of items sold. Use a large scale, as you will need to read values from your graph. From the graph, estimate the number of items she needs to sell each week to break even.

- Calculate the expenses and income for that number of items to check your estimate. Use your graph to estimate the number of items she needs to sell each week to make a profit of $1000 per week.
- Calculate the expenses and income for that number of items to check your estimate.
- Write an equation for income which shows how the number of dollars taken each week, ***m,*** is related to the number of items sold, ***n***.
- Write another equation for expenses which shows how the number of dollars spent each week, ***m,*** is related to the number of items sold, ***n***.

Answer #3

1) Draw up a table to show some values for income to the shop ***i,*** for different numbers of items sold, ***n***. Number of items sold ***n*** = 0, 100, 500, 1000, and 15000. If everything sells for $20, multiply ***n**** 20 to get income. For example, above, where there are 500 items, the income is 500 ***** 20 = 10000. Does this make sense? (income in dollars)

2) Draw up a table for the total cost for different numbers of items sold. These are the fixed costs added to the variable costs. Remember that the variable costs are directly related to the number sold. Number of items sold 0 = 100, 500, 1000,and 1500. If above, the variable costs are 7500 and there are 500 items, the variable costs are $7500/500 items. How much is that per item? Now can you figure out what the total variable costs are, depending on the items sold? (expenses in dollars)

3) The difference between expenses and income is the factory owner's profit. If she made no sales, she would still have to pay the fixed costs, so she would make a very large loss. If she made many sales she would have easily covered her fixed costs and made a good profit. In a week in which she sold 500 items did she make a profit or a loss? To find the profit/loss, here is what you do. Take income and subtract fixed cost ($3000/week) and variable cost (depends on number of items sold). Does this make sense? If you get a positive number, that is the profit. If it's negative profit, it's also called loss, during a week in which she sold 1000 items. Did she make a profit or a loss? On the same pair of axis draw the two graphs to show income related to the number of items sold and expenses related to the number of items sold. Use a large scale as you will need to read values from your graph. You need ***$*** on one axis and items sold on the other axis. Now you can plot two different sets of points: One is items sold versus variable expenses. For example, one point would be (500, 10500) since 7500 + 3000 = 10500. These are the numbers in your first table with 3000 added to each of the costs.

Answer #3 continued

The other is items sold versus income – for example, one point would be (500, 10000). Basically, these are just the numbers in your second table from the graph estimate the number of items she needs to sell each week to break even. Calculate the expenses and income for that number of items to check your estimate. Where do incomes equal expenses? This is the "break-even point."

Question #4

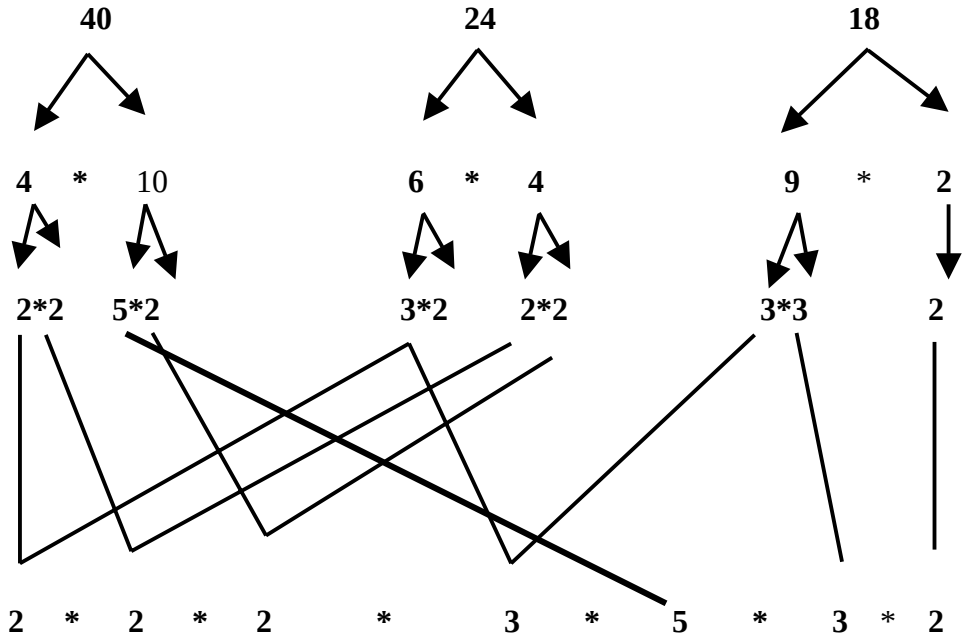
The Wachusett Inn received a shipment of plates packed in full cartons of 40 plates each. Another shipment of plates packed 24 to a carton went to the Sunnyside Restaurant. Plates were also shipped to Massachusetts University, but this shipment contained cartons with 18 plates each. If the hotel, the restaurant, and the university each received the same number of plates, and if none of them received more than a thousand plates, how many plates were in each shipment? How many cartons were in each shipment?

Answer #4

However many plates the Wachusett Inn received, the number must have been a multiple of 40, since they must have received a whole number of boxes. Likewise, the number of plates received by the Sunnyside Restaurant had to be a multiple of 24, and the number received by the University had to be a multiple of 18. If these three places received the same number of plates, call the number ***n***, the ***n*** must be a multiple of 40, of 24, and of 18. In this situation, ***n*** is called a common multiple of 40, 24, and 18. You are asked to find the least common multiple, or the smallest number, which is a multiple of all three of the numbers 40, 24, and 18.

Answer #4 continued

One way to do this is to factor the three numbers into prime factors:

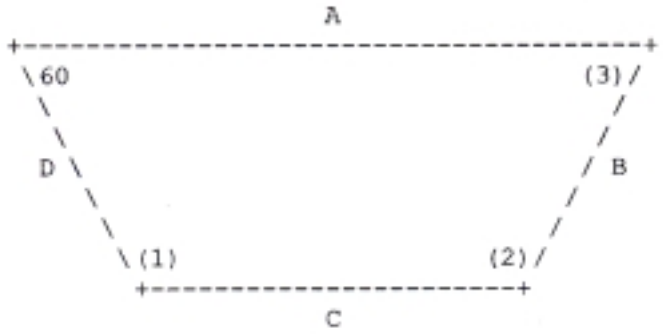


Once this is done, one can connect one pairs of the same prime number that come from different trees. For each pair connected, write the number once. Whatever is left is written once. These all get multiplied together (2*2*2*3*5*3*2) to get the common multiple, which is 720.

To find the number of cartons in each shipment, you need to divide 720 (the total number of glasses shipped to each location) by the number of glasses per carton shipped to that particular location. So, for instance, the number of cartons shipped to the Wachusett Inn would be 720/40 = 18 cartons. The cartons shipped to Sunnyside Restaurant would be 720/24 = 30 and the cartons shipped to Massachusetts University is 720/18 = 40.

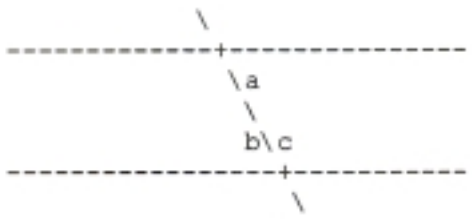
Question #5

If one angle of an isosceles trapezoid is 60 degrees, what are the measures of the other 3 angles?



Answer #5

In a trapezoid, sides ***A*** and ***C*** are parallel; in an isosceles trapezoid, not only are sides ***B*** and ***D*** equal, but angles 1 and 2 are equal. The 60 degree angle and angle 3 are also equal. For parallel lines intersected by a transversal, angles a and b are equal,



and angles b and c are supplementary (they add up to 180).
From this you can get (1) because a **=** 60,
so b equals 60 and ***b + c*** **=** 180, so c **=** 120°.
Next you can get (2) because 1 and 2 are the same.

Question #6

By what percent did the price of butter increase from 1936 to 1957 if in 1936 the butter cost 17 cents and in 1957 the butter cost 39 cents?

Answer #6

Make an estimate. If the price doubles that's a 100 percent increase.
So if it went from 17 to 34 it would be 100 percent.
So, by common sense, we can guess the answer will be about
110 percent because it went to 39 cents, not 34.

Here's the formula for finding percent increase:

$$\frac{\text{(Finishing price - Starting price)}}{\text{Starting price}} \times 100 = \text{Percentage Increase}$$

$$\frac{39 - 17}{17} \times 100 = 129.41\%$$

So our original estimate was off by a little, but was still fairly close.
Estimating is very useful!

Question #7

Janine's aunt is 60. She is 12 years older than twice Janine's age.
How old is Janine?

Answer #7

Use a variable to represent Janine's age, so let Janine's age = ***a***. We know from the information given in the problem that 60 is 12 more than twice Janine's age. Since Janine's age is a, twice Janine's age is 2***a***. 12 more than twice Janine's age is 12 + 2***a***. This equals 60 so we can make an equation with one variable that we can solve in order to get a: 60 = 12 + 2***a***.

- 60 = 12 + 2***a*** (original equation)
- 60 – 12 = 12 – 12 + 2***a*** (subtract 12 from both sides)
- 48 = 2***a***
- 48/2 = 2***a*** / 2 (divide both sides by 2)
- 24 = ***a***

So, Janine's age is 24.

Question #8

One day Mr. Davis told the class that he had just celebrated his birthday. The students all tried to guess his age. He gave these clues:

a) Mr. D's age divided by 5 leaves a remainder of 4.

b) Mr. D's age divided by 4 leaves a remainder of 3.

c) D's age divided by 3 leaves a remainder of 2.

d) D's age divided by 2 leaves a remainder of 1.

e) D's age is the smallest number that fits the other clues.

Answer #8

a) 59/5 **=** 11 ***R*** 4

b) 59/4 **=** 14 ***R*** 3

c) 59/3 **=** 19 ***R*** 2

d) 59/2 **=** 29 ***R*** 1

e) I looked for a prime number that was four more than a multiple of five.

The answer is 59.

Question #9

When Mr. I. M. Winner won the lottery, he decided that he would split the money among his four children. He would give 50 percent of his winnings to his oldest daughter, Crystal, who had just bought a new car and could really use the money. Another 25 percent of his winnings would go to his daughter, Ophelia, who would soon be finishing school. His son, Grant, would get 20 percent of the winnings while his youngest child, Jason, would get $700.

Determine the amount of money that Mr. I. M. Winner won in the lottery.

Answer #9

- $100\% - (50\% + 25\% + 20\%) = 5\%$
- Grant gets the remaining 5%, which is $700.
- $\frac{\$700}{x} = \frac{5\%}{100}$
- Cross multiply to get: $5x = 70{,}000$
- Divide both sides by 5 to isolate x: $\frac{5x}{5} = \frac{70000}{5}$

  $x = \$14{,}000$

Question #10

A disk jockey plays only Rap and Rock CDs. If the disk jockey plays 4 rap CDs for every 7 Rock CDs he plays, then

a) What fraction of the CDs he plays are Rock?

b) What percent of the CDs he plays are Rap?

c) Round the percent to the nearest whole number.

Answer #10

a) 7/11

b) 4/11 **=** 0.3636 ***x*** 100 **=** 36.36 %

c) 36%

Question #11

A merchant raises the price of a $100 item by 20%. After finding that she could not sell the item at the higher price, she discounted it by 20%.

a) What is the new price of the item after it was raised by 20%?

b) What is final price of the item?

c) Explain the reasons for your computations in both **a)** and **b)**.

Answer #11

a) $100 * 20% = $20
$100 + $20 = $120

b) $120 – ($120 x 20%)
$120 – $24 = $96

c) For part **a)** I multiplied the original price by the percentage that the price increased. I added this to the original price to get the new price.

For part **b)** I took the new price of $120 and multiplied it by 20% to get the amount the price was dropped, $24. I subtracted this from the new price of $120 to get $96.

Question #12

At the Johnson Middle School all students either walk or take the bus to school. The ratio of students who walk to school to those who take the bus is 2:5.

a) If 120 students walk 1 mile to school, how many students are there in the school?

b) Calculate the number of students who take the bus.

Answer #12

a) For every 7 students in the Johnson Middle School, 2 walk and 5 take the bus. If 120 walk, set up a proportion as follows:

- $\frac{2}{7} = \frac{120}{x}$
- $2x = 840$
- Divide both sides by 2 to get:

b) $\frac{2}{7} = 120$ *so*, $\frac{1}{7} = 60$. $60 * 5 = 300$.

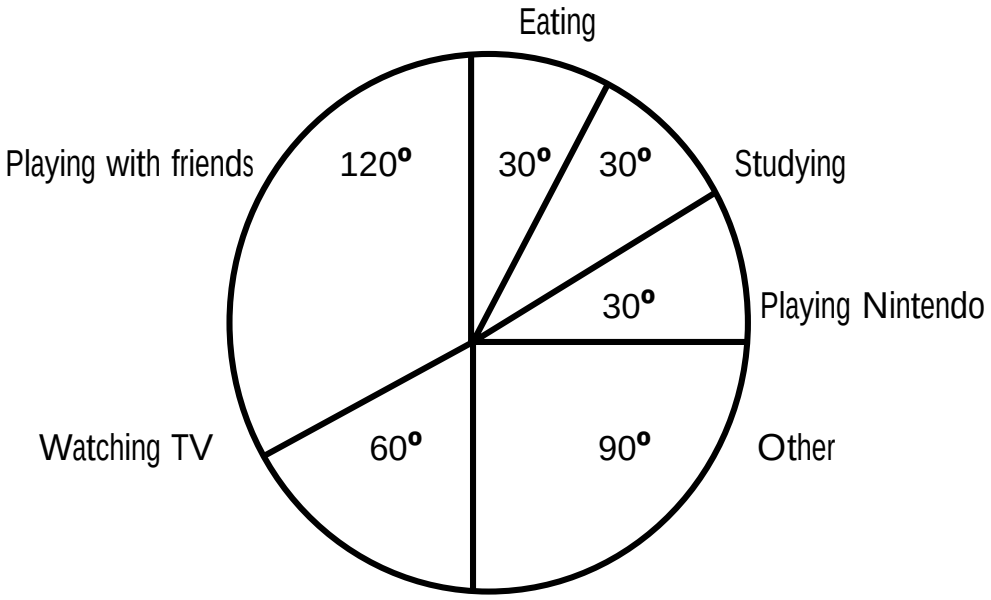
***x* = 420 total students in Johnson Middle School.**

Question #13

Pedro made a chart to record his activities between 9:00 AM and 9:00 PM on one Sunday. Fill in the blank circle graph from the data in Pedro's chart to show how he spent his time. Use percents in your graph.

Activities	Number of Hours
Eating	1
Watching TV	2
Playing Nintendo	1
Playing with friends	4
Studying	1
Other	3

Answer #13



This solution was found by adding the hours to find a total for the denominator of the fraction. Each activity became the numerator. Since fractions are division problems, these were divided in order to get a decimal equivalent. This decimal was multiplied by 360° in order to get the equivalent degrees.

For example:

- $\frac{3}{12} = \frac{x}{360}$
- **So,** $12x = 360 * 3$
- $12x = 1080$
- **Dividing both sides by 12 to get**

  $x = 90°$

Question #14

Pedro is twice as old as Maria. Three years from now, the sum of their ages will be 42. How old is Pedro?

Answer #14

Two-variable solution:

If we want to use two variables to express the given information, we will need two equations to solve for these variables.

Start by assigning variables. We want to find Pedro's age, so let's call that ***p***; but we need a variable for Maria's age too, so we will call her age ***m***.

We know that Pedro is twice as old as Maria. Another way of saying this is that Pedro's age is 2 times Maria's age. This gives us our first equation: $p = 2m$.

We also know that:

a) in three years the sum of Pedro's and Maria's ages will be 42;

b) in three years, Pedro's age will be 3 more than it is now, or $p + 3$;

c) the same is true of Maria's age: in three years; it will be $m + 3$.

Since the sum of the their ages in three years is 42, we have our second equation: $p + 3 + m + 3 = 42$.

Simplify by adding the numbers:	$p + m + 6 = 42$
Subtract 6 from each side:	$p + m = 36$

Answer #14 continued

Now we have two equations in two variables:

a) $p = 2m$

b) $P + m = 36$

Since Equation 1 provides an expression for p in terms of m that needs no simplification, we can plug the value for p in Equation 1 into the value for p in Equation 2: $2m + m = 36$.

add like terms:	$3m = 36$
divide both sides by 3:	$m = 12$

Now we know that Maria is 12 years old, which makes Pedro's age easy to find. All we need to do is plug $m = 12$ into either Equation 1 or Equation 2 and solve for p:

Equation 1	**Equation 2**
$p = 2m$	$p + m = 36$
$p = 2 \times 12$	$p + 12 = 36$
$p = 24$	$p = 24$

As we can see, Pedro is 24 years old. It doesn't matter which equation we use, since the value for Pedro's age must be the same in both cases.

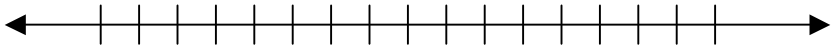
Again, it's always a good idea to check our answer.

a) Pedro is supposed to be twice as old as Maria. Pedro is 24; Maria is 12. Is 24 twice 12? Yes.

b) In three years, the sum of Pedro's and Maria's ages should be 42.
In 3 years, Pedro will be $24 + 3 = 27$ years old.
In 3 years, Maria will be $12 + 3 = 15$ years old.
Is the sum of 27 and 15 equal to 42? Yes.

We can see that we have found the correct answer.

Question #15



Use the number line above to answer the following questions:

a) Correctly position the following set of integers beneath the marks on the number line. **-3, +7, –2, –4, +6, +1**

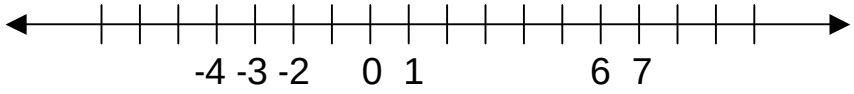
b) Explain why you decided where to place –2 on your number line.

c) Which number is greater: –2 or +6? Explain your answer.

d) Which number is greater: –2 or –4? Explain your answer.

Answer #15

a)



b) placed negative 2 on the number line by counting two spaces to the left of zero.

c) +6 is greater than –2 because +6 is to the right of –2 on the number line.

d) –2 is greater than –4. As you move to the right on the number line the value of the number becomes greater. Conversely, as you travel to the left on the number line, numbers get smaller.

Question #16

A farmer grows 252 kilograms of apples. He sells them to a grocer who divides them into 5 kilogram and 2 kilogram bags. If the grocer uses the same number of 5kg bags as 2kg bags, how many bags did he use in all?

Answer #16

x = total number of bags

$$5(1/2x) + 2(1/2\ x) = 252$$

$$2.5x + 1x = 252$$

$$3.5x = 252$$

$$x = \mathbf{72\ bags}$$

Question #17

Mrs. Pi gave a surprise math quiz. Below are the results of the quiz.

a) Create a stem leaf plot for this data.

b) Calculate the maximum, minimum, range, median, and mode. Show your work.

DATA: 85, 95, 85, 15, 80, 75, 100, 60, 70, 90, 85, 75

Answer #17

maximum = 100; minimum = 15; range = 85; median = 82.5; mode = 85

Stems	Leaves
1	5
2	
3	
4	
5	
6	0
7	0 5 5
8	0 5 5 5
9	0 5
10	0

Question #18

Write the following fractions in decimal form, rounding to the thousandth place.
Show your work. With this knowledge place the numbers in order from least to greatest.
Explain your reasoning.

$$\frac{1}{2} \qquad \frac{1}{3} \qquad \frac{3}{4} \qquad \frac{2}{5} \qquad \frac{2}{3}$$

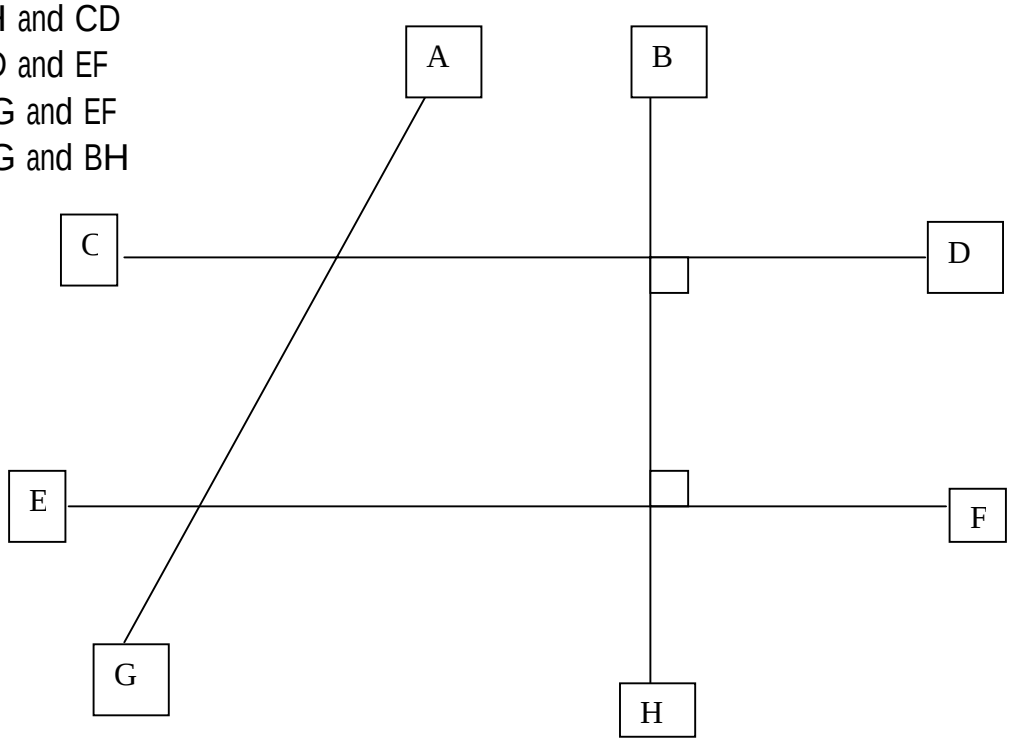
Answer #18

- $\frac{1}{2}$ = .500 or .5; $\frac{1}{3}$ = .333; $\frac{3}{4}$ = .750 or .75; $\frac{2}{5}$ = .400 or .4; $\frac{2}{3}$ = .667
- in order from smallest to greatest = $\frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

Question #19

From the following pairs of line segments, identify if the pair is parallel, perpendicular, intersecting, or skew. Explain your reasoning.

a) BH and CD
b) CD and EF
c) AG and EF
d) AG and BH



Answer #19

a) **parallel = CD & EF**

b) **perpendicular = BH & CD and BH & EF**

c) **intersecting = AG & CD; and CD & BH; and AG & EF; and BH & EF**

d) **skew = AG & BH**

Question #20

Farmer O. MacDonald needs to build a fence around his cattle pasture.
One pasture is shaped like a square that is 80 feet long on one side.

a) What is the total length of fence needed to enclose this pasture?

b) The other pasture is shaped like a rectangle 150 feet long and 40 feet wide. What is the total length of fence needed to enclose this pasture?

c) Which pasture has more area for the cattle to graze and why?

Answer #20

a) All the sides of a square are the same length; therefore, the perimeter is 4 **x** 80 feet = 320 feet.

b) The perimeter of the rectangle is (2***x*** length) + (2***x*** width) = (2 **x** 150 ft.) + (2 **x** 40 ft.) = 300 **+** 80 = 380 feet.

c) Area of a square **=** (length of a side)2 **=** (80 ft.) **=** 6400 sq.ft. Area of a rectangle **=** length x width **=** 150 ft. **x** 40 ft. **=** 6000 sq.ft.; therefore the area of the square is larger.

Question # 21

The Wachusett girl's basketball team had another win tonight. Jen scored $\frac{1}{2}$ of the points, Sarah scored $\frac{1}{12}$ of the points, and Katie scored $\frac{1}{3}$ of the points. Anna and Jessica scored the remaining 6 points, and each played a great defensive game. How many points did the team score? How many points did Jen, Sarah, and Katie score?

Answer #21

The team scored 72 points. Jen scored 36 points, Sarah scored 6 points, and Katie scored 24 points.

Let P = points.

- $\frac{1}{2}P + \frac{1}{3}P + \frac{1}{12}P + 6 = P$
- $12\ (\frac{1}{2}P + \frac{1}{3}P + \frac{1}{12}P + 6) = P * 12$
- $6P + 4P + P + 72 = 12P$
- $11P + 72 = 12P$
- $72 = P$

$\sim$

- Jen $= \frac{1}{2}P = \frac{1}{2} * 72 = 36$
- Sarah $= \frac{1}{12}P = \frac{1}{12} * 72 = 6$
- Katie $= \frac{1}{3}P = \frac{1}{3} * 72 = 24$

Question #22

Jeremy receives a weekly allowance of $12. He can earn $5 more for each special job he completes beyond the usual chores. In twelve weeks, Jeremy wants to have saved enough money to buy a skateboard that costs $250. How much money will he earn with his weekly allowance after twelve weeks? If he saves all of his money, how many special jobs does he need to complete to earn enough money to buy the skateboard? How many do you suggest Jeremy do each week to reach his goal?

Answer #22

- After twelve weeks, Jeremy earned $144 ($12/wk * 12 weeks).
- He needs an extra $106 ($250 – 144) to buy the skateboard.
- Jeremy earns $5 per special job. $106/$5 = 21.5, so he needs to complete 22 special jobs to earn enough money.
- Jeremy should perform about 2 extra jobs per week to reach his goal.

Question #23

American Eagle is having a huge sale! Channing went shopping and chose a pair of jeans and sweater that totaled $86. Channing has two coupons, one for $15 off a purchase of $75 or more, and another coupon for 20% off the entire purchase. Channing can only use one coupon. In order to pay the least amount, which coupon should Channing use? Explain your reasoning.

Answer #23

- Channing should use the 20% off coupon in order to pay the least amount for the outfit.
- Using the 20% off coupon, Channing will pay $68.50 for the outfit.
  - $86 * (1-.20) = $86 * .8 = $68.80
- Using the $15 off coupon, Channing would pay $71 for the outfit.
  - $86 – 15 = $71.00

Question #24

Chloe wants to travel to see her cousin, Kelly, who lives 1000 kilometers away. How many hours will it take Chloe to drive there if she travels at an average speed of 55 miles per hour?

Answer #24

- It will take Chloe 11.273 hours to travel 1000 kilometers.
  - 1000 km = _____ mi.
  - 1 km = .62 miles
  - 1000 km = .62 miles * 1000
  - 1000 km = 620 miles
  - 55 miles / hour = 1/55 hour / mile
- 620 miles * 1 hour / 55 miles = 11.273 hours

Question #25

How many blocks, 3cm on each edge, can be stored in a box 12cm on each edge? What happens to the volume of the box if the length of an edge is doubled to 24cm?

Answer #25

- ***e*** **=** 12cm divided by 3cm blocks **=** 4 blocks
- ***V*** **=** ***e*****^** 3 **=** (4 blocks) **^** 3 **=** 4 ***** 4 ***** 4 **=** 64 blocks

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- ***e*** **=** 24cm ÷ 3cm blocks **=** 8 blocks
- ***V*** = ***e*** **^** 3 **=** (8 blocks) **^** 3 = 8 ***** 8 ***** 8 **=** 512

The volume of the box with an edge double the length of the previous box is increased 8 times.

Question #26

Sarah is making a sandwich. Her bread choices are wheat or white bread. Her meat choices are turkey, roast beef, or ham. Her condiment choices are mustard, mayonnaise, or no condiment.

How many combinations of sandwiches are possible? Make a tree diagram to show all of Sarah's sandwich possibilities. If she chooses randomly, what is the probability that her sandwich will have no condiment on it?

Answer #26

Sarah can make 18 sandwich combinations.
She has 2 bread choices * 3 meat choices * 3 condiment choices = 18

The probability that Sarah's sandwich has no condiment on it is 1/3.
2 bread choices * 3 meat choices * 1 no condiment = 6/18 = 1/3

• white bread • turkey	• turkey with mustard on white • turkey with mayo on white • turkey no condiment on white
• white bread • roast beef	• roast beef with mustard on white • roast beef with mayo on white • roast beef no condiment on white
• white bread • ham	• ham with mustard on white • ham with mayo on white • ham no condiment white
• wheat bread • turkey	• turkey with mustard on wheat • turkey with mayo on wheat • turkey no condiment wheat
• wheat bread • roast beef	• roast beef with mustard on wheat • roast beef with mayo on wheat • roast beef no condiment on wheat
• wheat bread • ham	• ham with mustard on wheat • ham with mayo on wheat • ham no condiment on wheat

Question #27

Farmer Fred bought 24 meters of fencing to make a coop for his chickens. How many different rectangular coops could Farmer Fred make? Make a list of all the possibilities. Which one would provide the largest area for the chickens? The lengths of the sides have to be whole numbers.

Answer #27

Farmer Fred could make six different rectangular coops using 24 meters of fencing.

The different coop dimensions are:

Coop #1	***A*** = 11m **x** 1 ***m*** **=** 11 square meter ***P*** **=** 2(11) **+** 2(1) **=** 24 meters
Coop #2	***A*** **=** 10***m*** **x** 2 ***m*** **=** 20 square meter ***P*** **=** 2(10) **+** 2(2) **=** 24 meters
Coop #3	***A*** **=** 9m **x** 3 m **=** 27 square meters ***P*** **=** 2(9) **+** 2(3) **=** 24 meters
Coop #4	***A*** **=** 8m **x** 4***m*** **=** 32 square meters ***P*** **=** 2(8) **+** 2(4) **=** 24 meters
Coop #5	***A*** **=** 7***m*** **x** 5m **=** 35 square meters ***P*** **=** 2(7) **+** 2(5) **=** 24 meters
Coop #6	***A*** **=** 6***m*** **x** 6***m*** **=** 36 square meters ***P*** **=** 2(6) **+** 2(6) **=** 24 meters

Using 24 meters of fencing, Farmer Fred should make a 6-meter by 6-meter coop, which will provide the most area of 36 square meters.

Question #28

Over the term, Katie scores 86, 92, 78, 82, and 71 on her five math tests. What is the average of Katie's test scores? What is the range of her test scores?

Katie has one more test to take this term. Given that grades can range from 0 to 100 on tests, what is the lowest Katie can average for all six tests? What is the highest she can average for all six tests? Can you determine what score she needs to get on the test in order to have a final term average of an 83?

Answer #28

- The average on the five tests is an 82. ((86 + 92 + 78 + 82 + 71)/5 = 82)
- The range of her five test scores is 21 points (92 – 71)
- Her lowest possible average is a 68. (86 + 92 + 78 + 82 + 71 + 0) / 6 = 68
- Her highest possible average is an 85 (86 + 92 + 78 + 82 + 71 + 100) / 6 = 85
- In order to have a final average of an 83, she needs to score an 89 on her next test.
  - (86 + 92 + 78 + 82 + 71 + x) / 6 = 83
  - 86 + 92 + 78 + 82 + 71 + x = 83 * 6
  - 409 + x = 498
  - x = 498 – 409
  - x = 89

Question #29

Susie's class is having a pizza party. Susie volunteered to bring the soda. There are 20 students in the class and each student will drink an 8 fluid ounce cup of soda. How many 2-liter bottles of soda does Susie need to bring to the party to have enough soda for everyone?

Answer #29

In order to have 8 fluid ounces of soda per student, Susie needs to bring 3 bottles of soda to the party.

1 liter = 1.06 quarts → 2 liters = 2.12 quarts	↓ 1 quart = 2 pints 2.12 quarts = 4.24 pints
	↓ 1 pint = 2 cups 4.24 pints = 8.48 cups
	↓ 1 cup = 8 fluid ounces 8.48 cups = 67.84 fluid ounces

- 20 students x 8 fluid ounces per student = 160 fluid ounces needed
- 160 ounces ÷ 67.84 ounces per 2-liter bottle ≈ 2.4, or 2-liter bottles of soda
- Since you can't bring a part of a bottle of soda, you need to round the 2.4 bottles of soda to 3 bottles of soda.

Question #30

The gauge of an oil tank indicated that the tank was $\frac{1}{7}$ full. After 280 gallons of oil were added to the tank, the gauge indicated that the tank was $\frac{5}{7}$ full. How many gallons of oil does the tank hold, assuming the gauge is accurate?

To solve the problem, write an algebraic expression. Then solve the expression.

Answer #30

- The oil tank holds 490 gallons of oil.
- x = the amount oil the tank holds
  - $\frac{1}{7}x + 280 = \frac{5}{7}x$
  - $-\frac{1}{7}x + \frac{1}{7}x + 280 = \frac{5}{7}x + -\frac{1}{7}x$
  - $280 = \frac{4}{7}x$
  - $\frac{7}{4} * 280 = \frac{4}{7}x * \frac{7}{4}$
  - $7 * 70 = x$
  - $490 = x$

Question #31

John had $90 in her saving account. He then received $75 for a present and earned an additional $50 from mowing laws. He went shopping and bought 2 CD's with some of the money and also spent $21 for a shirt. He went to another store and bought an additional CD. When he got home he had $152 left. If the CD's cost the same at both stores, what was the cost of a CD?

Answer #31

- Total amount of money that John had was $90 + $75 +$50 = $215.
- Total money spent was $21 + 3* ***P***, when ***P*****=** cost of one CD
- Money left = amt of money had – money spent = $152
- $152 = $215 – ($21 + 3***P***) = $215 – $21 – 3***P*** = $194 – 3***P***
- 3***P*** = $194 – $152 = $42
- ***P*****=** $42/ 3 = $14 per CD

Question #32

Abe, Betty, Carl, Dennis, and Emily are all going to be participating in a fund-raising walk-a-thon. At the conclusion, Betty had walked 2 miles more than Abe. Carl walked half as many miles as Betty and Dennis walked 3 miles less than Abe. Emily walked 6 more miles than Dennis. If the students got $4 per mile walked, write an equation that shows the amount of money that each student raised based on the distance that Abe walked.

Answer #32

Distance	***Dollars***
A **=** distance Abe walked	Abe raised 4***A*** dollars
↓	↓
A **+** 2 = distance Betty walked	Betty raised 4 (***A*** **+**2) **=** 4***A*** **+** 8 dollars
½ ***A*** **+** 2 = distance Carl walked	Carl raised 4 (1/2 (***A*** **+**2)) **=** 2 (***A*** **+**2) = 2***A*** **+**4 dollars
A **–** 3 = distance Dennis walked	Dennis raised 4 (***A*** **–** 3) **=** 4***A*** **–** 12 dollars
(A – 3) + 6 = distance Emily walked	4 ((***A*** **–** 3) **+** 6) **=** 4 (***A*** **+** 3) **=** 4***A*** **+** 12 dollars

Question #33

Students in Mr. Sanford's are making wall decorations for the Jr. High dance. They had 15 that were left over from trial runs. Below is the schedule for how many need to be done after each day of work.

Day	1	2	3	4	5
Decorations	35	55	75	95	105

Question #33 continued

- Write a mathematical expression to indicate the number of decorations that will be available at the end of any particular day.
- How many decorations will be completed at the end of 11 days?
- How many days will be needed to make 360 decorations?

Answer #33

D = the number of days, therefore the number of decorations is equal to:

- $n = 20d + 15$
- After 11 days, $n = (20 \times 11) + 15 = 235$
- If you need 360 decorations, you need to solve for days, so:
  - $360 = 20 \times d + 15$, so $345 = 20 \times d$, so
  - $d = 345/20 = 17.25$ days

Question #34

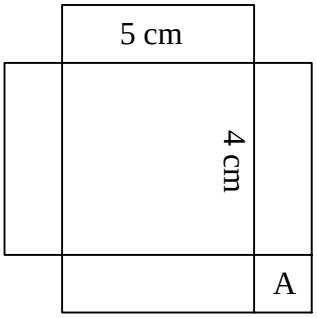
Inside a box there were 12 pieces of paper with the numbers 1 to 12 on them.
Five students drew out 2 pieces of paper and the added the totals together.
Al's total was 11, Beth's was 4, Cal's was 16, Debby's was 7, and Eliza's was19.
What were the numbers picked by each student?

Answer #34

- Al – 4 & 7
- Beth – 1 & 3
- Cal – 6 & 10
- Debby – 2 & 5
- Eliza – 8 & 11

Question #35

A rectangular sheet of wood has four small squares removed.
It is then cut to make a box that is 5cm by 4cm with a volume of 60cm^3.
(Four pieces of size A4 are removed.)



Find the original area of the sheet of wood.

Answer #35

- The volume of the box is (length) **x** (width) **x** (height) and is equal to $60cm^3$.

- If we fold up the sides, we can substitute into the equal line length (5cm) and the width (4cm).

  $60cm^3 = 5cm \times 4cm \times height$
  $height = 60cm^3/20cm^2 = 3cm$

- The height is equal to the length of the sides ***"A,"*** so the overall length of the piece of wood is $5cm + 2(A) = 5 \times 2(3) = 11cm$, and the width of the piece of wood is $4cm + 2(A) = 4 \times 2(3) = 10cm$;

  therefore, the area of the original sheet of wood is
  $A = l \times w = 11cm \times 10cm = 110cm^2$.

Question #36

The football stadium at Crestview Middle School has a seating capacity of 600. For the first game, $\frac{1}{3}$ of the seats were empty. For the second game, 85% of the seats were filled and for the third game, there were 500 fans in attendance. Which game had the greatest attendance? What was the total attendance and the average attendance for the three games?

Answer #36

- For the first game, $\frac{2}{3}$ of the seats were occupied, so

  $$\#\text{ of spectators} = \frac{2}{3} \times 600 = 400 \text{ spectators.}$$

- For the second game, # of spectators = $0.85 \times 600 = 510$ spectators. $510 > 500 > 400$; therefore there were more spectators at the second game.

- Total attendance = $400 + 510 + 500 = 1410$ spectators.

- Average attendance = total attendance/# of games = $1410/3 = 470$ spectators.

Question #37

1) Arrange the following values in order from smallest to largest:

$0.5,\ \frac{7}{8},\ 0.40,\ 0.75,\ \frac{4}{5},\ 0.66,\ \frac{1}{3}$

2) Tony has a 20% off coupon for a CD at Rock-A-Lot Music store. If he finds a CD that he likes for $8.00 and has only $7.00 with him, does he have enough money to buy the CD? If so, how much money will he have after he buys the CD?

3) If a golf pro has 21 golf balls and he is putting them into boxes that hold a maximum one-half dozen, how many boxes will he need to all the balls into a box?

Answer #37

a) $\frac{1}{3} < 0.40 < 0.5 < 0.66 < 0.75 < 4/5 < \frac{7}{8}$

b) Yes, and he'll have $0.60 left.

c) He'll need 4 boxes. (The last one will only have 3 balls in it, but he still needs the box.)

Question #38

a) Members of the 6th grade play were given 24 tickets to sell. Jamie sold $\frac{2}{3}$ of his tickets, Kim sold 50% of her tickets, Matt sold 0.75 of his tickets, and Sally sold $\frac{7}{12}$ of her tickets. Who sold the most? How many total tickets were sold?

b) Jackie has decided to save $8 per week to buy a CD player that costs $115. What is the mathematical expression that will allow her to calculate how much she will still have to save after n-number of weeks?

Answer #38

a) Joanne – 2/3 **x** 24 = 16 tickets.
Kim – 50% = ½, so ½ **x** 24 = 12 tickets.
Matt – .75 **x** 24 = 18 tickets
Sally – 7/12 **x** 24 = 14 tickets

Matt sold the most tickets (18)

Total tickets sold = 16 + 12 + 18 +14 = 60 tickets

Answer # 38 continued

b) Jackie is going to have $8 per week. If she saves for ***"n"*** weeks, the amount she has saved after any number of weeks is $8 \cdot n = 8n$,

therefore, the amount left that she needs is the original cost of the CD player, $115, minus the amount she has saved, or $115 – 8*n*.;

Question #39

Midas collected his money and stacked it up in piles one mile high. He had one pile of dollar bills, one pile of dimes, and one pile of quarters. He found that a dollar bill was 0.004 inches thick. He also determined that a dime was 14 times as thick as a dollar and a quarter was 16 times as thick as dollar.

- How much money did he have?
- If he had a million dollars in quarters, about how high would that pile be?

Answer #39

- Thickness of a dime = 0.004 x 14 = 0.056 inches.
  Thickness of a quarter = 0.004 x 16 = 0.064 inches.
- Convert 4 miles into inches:
  1 mile x 5280 ft./mile x 12 inches/ft. = 63,360 inches.

Answer #39 continued

- The number of dollar bills $= \frac{63360 \text{ inches}}{0.004 \text{ inches/bill}} = \$15{,}840{,}000$

  The number of dimes $= \frac{63360 \text{ inches}}{0.064 \text{ inches/dimes}} \cong 1{,}131{,}427.6 \text{ dimes}$

- To find the value of the dimes, divide the number of dimes by 10 (10 dimes equal one dollar) to get the dollar amount of $113,162.86.

- The number of quarters $= \frac{63360 \text{ inches}}{0.064 \text{ inches/quarter}} = 990{,}000 \text{ quarters}$

  To find the value of the quarters, divide the number of quarters by 4 (4 quarters equal on dollar) to get the dollar amount of $247,500.

- Total ≅ $15,840,000 + $113,142 + $247,500 ≅ $16,200,600

- We know that $247,500 in quarters is 1mile high, so the height of $1,000,000 in quarters is equal to $1,000,000/$247,500/mile ≅ 4 miles.

Question #40

A shipping company charges the following rates based on weight (lbs) for shipping a package: 1 lb costs $2.60, 2 lbs costs $3.50, 4 lbs costs $5.30. Predict the cost to ship a 5 lb. package and a 10 lb. package. Develop a mathematical expression to predict the cost of shipping any package.

Answer #40

A 5 lb. package costs $6.20, and a 10 lb. package costs $10.70 to ship.
Cost = $2.60 + ($0.90 * total weight, –1 lb.)

Question #41

Tom has 100 cards and he has them numbered 1-100. Starting with number 1, he places an ***"X"*** on every 5th card, an "O" on every 7th card and a ***"$"*** on every 10th card. What number card will have all three symbols (***"X,O,$"***) on it?

Answer #41

There are two ways to solve this problem:

(1) Set up a series of 100 cards and follow the directions.

(2) Determine first number that is divisible by 5, 10, and 7. (the easier way to solve this problem).

Number 70 is the first number that is divisible by 5, 10 & 7; therefore the answer is card #70.

Question #42

a) Jim has decided to save $1.25 a day to buy a CD player that costs $55. What is the mathematical expression that shows how much he will have to save after "n" weeks?

b) Monica is making a scale model of the Capital State Building in her state. The height of the building is 984 feet and the area of the square base is 108,900 square feet. If the model's height is 12 inches, what is the length of one side of the base.

Answer #42

a) If Jim saves $1.25 each day, his weekly total will be 7 (# of days per week) **x** $1.25.
"n" equals the number of weeks, therefore, n x 7 **x** $1.25 equals amount Jim saved after n weeks.

If the CD player costs $55, the amount Jim would need after ***"n"*** weeks would be $55 – (***n*** **x** 7 **x** $1.25) **=** $55 **–** $8.75 ***n.***

b) Find the length of one side of the base of the Capitol State Building. Since the area of a square equals the length of one side squared, the length of one side equals the square root of the area, or
length **=** $\sqrt{108{,}900 \text{ sq.ft.}}$ **=** 330 feet.

Set up a ratio of actual size to model size:

$$\frac{984 \text{ ft.}}{12 \text{ inches}} = \frac{330 \text{ ft.}}{x \text{ inches}} \quad x = 330 \text{ ft. } x \frac{12 \text{ inches}}{984 \text{ ft.}} \quad x \cong 4 \text{ inches}$$

Question #43

Kelsey and Geoffrey rode the Roller coaster three times more than they rode the Whip. They rode the Whip one more time than they rode the Merry-Go-Round. They rode the Merry-Go-Round four times. How many times did they ride each ride?

Answer #43

- Students work backwards to solve the problem. They know that Kelsey and Geoffrey rode the Merry-Go-Round 4 times. Since they rode the Whip one more time than the Merry-Go-Round, 4 + 1 = 5.

Answer #43 continued

- They rode the Roller Coaster 3 times more than the Whip:
  3 x ***W*** = ***R*** (3 x 5 = 15).
- Kelsey and Geoffrey rode the Merry-Go-Round 4 times, the Whip 5 times, and the Roller Coaster 15 times.

Question #44

Use the following table to answer the questions below:

Coin Denomination	PENNY	NICKEL	DIME	QUARTER	HALF DOLLAR	SILVER DOLLAR
Weight in grams	2.500 grams	5.000 grams	2.268 grams	5.670 grams	11.340 grams	8.100 grams
Diameter in inches	0.750 in.	0.835 in.	0.705 in.	0.955 in.	1.205 in.	1.04 in.
Diameter in millimeters	19.05 mm	21.21 mm	17.91 mm	24.26 in.	30.61 mm	26.50 mm
Thickness	1.55 mm	1.95 mm	1.35 mm	1.75 mm	2.15 mm	2.00 mm

Source: United States Mint

Question #44 continued

1) Arrange the coins by weight from the greatest to the least.

2) Arrange the coins by thickness (mm) from the least to the greatest.

3) Arrange the coins by diameter (inches) from the least to the greatest.

4) How many pennies do you need to equal the weight of one nickel?

5) How much more does a silver dollar weigh than 4 dimes?

6) How much more does a silver dollar weigh than one quarter?

7) Patrick has 3 quarters, 4 pennies, 2 nickels and 1 silver dollar in his pocket. What is the total weight of the coins?

8) Nina has 2 silver dollars, 5 dimes, 1 quarter, and 2 nickels in her backpack. What is the total weight of the coins?

Answer #44

1) 11.340, 8.100, 5.670, 5.000, 2.500, 2.268

2) 1.35, 1.55, 1.75, 1.95, 2.00, 2.15

3) 0.705, 0.750, 0.835, 0.955, 1.04, 1.205

4) 2 pennies (2.5 + 2.5 = 5.0)

5) 1.296 grams (8.100 grams – 3(2.268 grams) = 1.296 grams)

6) 2.430 grams (8.100 – 5670 = 2.430 grams)

7) 45.100 grams (3(5.670) = 4(2.500) + 2(5.000) = 8.100 = 45.100 grams

8) 43.210 grams (2(8.100) + 5(2.268) + 5.670 + 2(5.000) = 43.210 grams

Question #45

The Central Tree Middle School basketball team is playing Paxton Center School. The game is tied 61-61. In his excitement, the Paxton Center coach steps onto the court just as the buzzer sounds and a technical foul is called. The Central Tree coach has to choose one of his players to shoot the free throw. If the player makes the free throw, Central Tree wins the game. The Central Tree coach has three players to choose from to shoot the free throw. In the pre-game warm-ups:

- Ashley made 17 out of 25 free throws
- Jessica made 15 out of 20 free throws
- Megan made 7 out of 10 free throws

Which player should the Central Tree coach select to shoot the free throw? Explain your reasoning.

Answer #45

1) We should first convert each of the players' free throw statistics into a fraction and then each of these into a percent. Ashley's probability was 17/25 or 68%. Jessica's probability was 15/20 or 75%, and Megan's probability was 7/10 or 70%.

2) By converting each of these statistics into percents we can see that Jessica would be the best person to choose for the free throw, for she made the highest percentage of free throws taken before the game.

Question #46

Peggy has one piece of butter that measures $\frac{1}{8}$ cup and another piece that measures $\frac{1}{4}$ cup. The cake recipe calls for $\frac{3}{4}$ of a cup. How much butter does she have? Does she have enough to make a cake? Explain your answer.

Answer #46

Peggy has $\frac{3}{8}$ cup and she needs $\frac{6}{8}$ cup. She does not have enough butter to make the cake. You need to add $\frac{1}{8}$ cup of butter with $\frac{1}{4}$ cup of butter but that can only be done by making the denominators the same. Change $\frac{1}{4}$ to $\frac{2}{8}$ and then add $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$. If we change $\frac{3}{4}$ to an equivalent fraction by multiplying the numerator and denominator by 2 that equals $\frac{6}{8}$ we can see that we have $\frac{3}{8}$ cup of butter and need $\frac{6}{8}$ cup of butter.

Question #47

USE THE TABLE TO SOLVE THE QUESTIONS BELOW:

Berries	Weight	Calories	Vitamin C (milligrams)
Blackberries	3 ½ ounce	52	21
Blueberries	3 ½ ounces	56	13
Cranberries	3 ½ ounces	49	14
Raspberries	3 ½ ounces	49	25
Strawberries	3 ½ ounces	30	57

Katrina ate $3\frac{1}{2}$ ounces of berries every day. She ate blackberries on Monday, Blueberries on Tuesday, cranberries on Wednesday, raspberries on Thursday, and strawberries on Friday.

a) How many pounds of berries did she eat in all?

b) How many calories on average did Katrina eat everyday.

c) How many milligrams of Vitamin C did she get on average everyday?

Answer #47

a) $17\frac{1}{2}$ ounces = 1 pound and $1\frac{1}{2}$ ounces

b) 47.2 calories

c) 26 mg.

Question #48

Mrs. Hackett has a 6 m long and 4 m wide rectangular garden.
She wants to plant flowers on half of the garden.

a) What is the area of the garden?

b) Draw it on the paper and shade in the part that would be flowers with one of the colors.

c) What is the area of the half of garden?

Answer #48

a) 24 sq.in. (6 x 4 = 24 sq.in.)

b) diagram

c) 12 sq.in. (24/2 = 12)

Question #49

Brian made 4 different kites. He wanted to make them all the same size.
When he measured the lengths of the kites, he came up with the following number:

Kite #1	$3\frac{3}{6}$ feet
Kite #2	$3\frac{4}{6}$ feet
Kite #3	$3\frac{2}{3}$ feet
Kite #4	$3\frac{1}{3}$ feet

Question #49 continued

a) Which two kites were of the same length?

b) Which kite was the shortest?

c) Which kite was the longest?

d) What was the difference in length between the longest and the shortest kites?

Answer #49

a) Kites 2 and 3

b) Kite 4

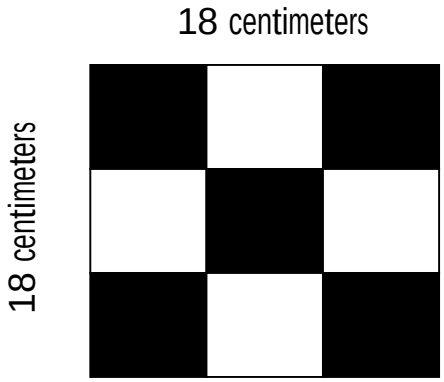
c) Kites 2 and 3

d) $\frac{1}{3}$ feet

Question#50

Kelsey wanted to add a patch to her mother's quilt.
She decided to add a design called a Nine-Patch.
It's geometric shape was a square with sides 18 centimeters long. She was to use two different colors of fabric, black and white, and needed to know how much cloth of each color she needed for the Nine-Patch.

Answer #50



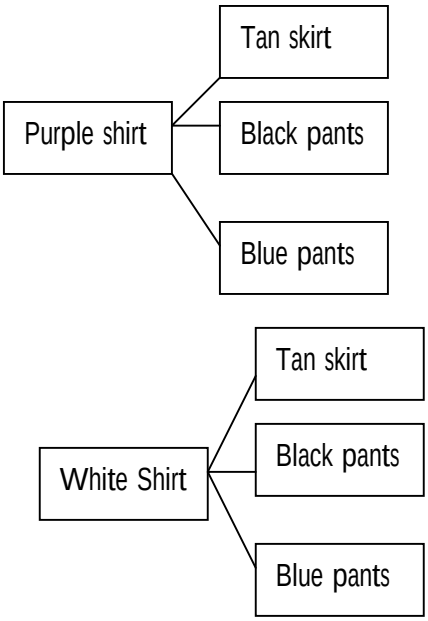
- First Kelsey decided on the Nine-Patch.
- The she computed the area of the large square:
  - A = 18 centimeters x 18 centimeters
  - A = 324 square centimeters
- Next she computed the area of each of the small patches:
  - A = 6 centimeters x 6 centimeters
  - A = 36 square centimeters
- Then she decided on 5 small black patches and 4 small white patches.
- Area for 5 small black patches:
  - A = 36 square centimeters x 5
  - A = 180 square centimeters:
- Area for 4 small white patches:
  - A = 36 square centimeters x 4
  - A = 144 square centimeters
- Kelsey needs 180 square centimeters of black fabric and 144 square centimeters of white fabric.
- To check the answer180 sq centimeters + 144 sq centimeters = 324 square centimeters which is the area of the entire large square.

Question #51

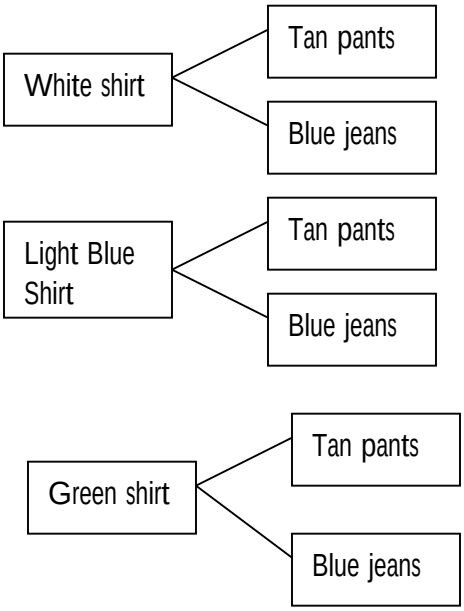
It was time for back-to-school shopping. Carly and Deidre were twins, and they were going shopping with their mom. They spent 7 hours at the mall and went into 11 different stores. Carly picked out a tan skirt, black pants, pair of blue pants, a purple shirt, and a white shirt. She spent all but $2.00 of her money. Deidre took her mom's advice and bought things that coordinated well with each other. She bought a white shirt, a light blue shirt, and a green shirt. She also bought a pair of tan pants and a pair of blue jeans. She spent all but $1.00 of her money. After shopping, the two girls compared their purchases, and each of them **predicted** that they could make more combinations with their clothes. How many combinations can they each make? Who can make more?

Answer #51

Carly's Tree Diagram



Deidre's Tree Diagram



Carly's results: 6 combinations

Deidre's results: 6 combinations

Both Carly and Deidre can make the same number of combinations, which is 6.

Question #52

Shamus and Owen went to the movies and got there early. Looking around the theatre Shamus realized that the shape of the movie theater is like a parallelogram. It gets wider as you get farther away from the screen. The first row has 9 seats and there are 33 rows in the theatre. I wonder how many seats there are in the last row compared to the first row.

Answer #52

- There are 9 seats in row 1.
- Variables:
  - x = number of seats in each row
  - y = row number

Row 1	Row 33
$x = 9 + (y - 1)$	$x = 9 + (y - 1)$
$x = 9 + (1 - 1)$	$x = 9 + (33 - 1)$
$x = 9 + 0$	$x = 9 + 32$
$x = 9$	$x = 41$

There are 41 seats in the last row compared to 9 in the first row.